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TIME TO RECRUITMENT FOR A TWO GRADE MANPOWER SYSTEM WITH TWO SOURCES OF DEPLETION AND CORRELATED INTER-POLICY DECISIONS TIMES USING UNIVARIATE MAX POLICY OF RECRUITMENT

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ABSTRACT
In this paper a two graded manpower system in which depletion occurs due to policy and transfer decisions which form respectively two independent sources of depletion is considered. A stochastic model is constructed and using a univariate Max policy of recruitment, the variance of time to recruitment is obtained when inter-policy decision times form a sequence of exchangeable and constantly correlated exponential random variables. Analytical results for the variance of the time to recruitment and other related performance measures.

KEYWORDS: Two grade manpower system, two sources of depletion of manpower, exchangeable and constantly correlated inter-policy decision times, univariate Max policy of recruitment, and Variance of the time to recruitment.

INTRODUCTION
Depletion of manpower occurs whenever announcing a new policy decisions (this form one source for depletion) in any organization. Transfer decisions which are non-recurrent in nature may also lead to depletion of manpower, thereby forming a second source for depletion. Frequent recruitment is costlier and hence recruitment is postponed to a point called threshold, a maximum allowable loss of manpower, beyond which the organization cannot run as usual. Consequently, a suitable decision on making recruitment is to be designed in order to offset the depletion of manpower. Elangovan et.al [6] have initiated the study on recruitment problem for a single grade manpower system with **two sources of depletion** and obtained the variance of time to recruitment using univariate CUM policy of recruitment when the loss of man power in the organization due to the two sources of depletion, inter-policy decision times, inter-transfer decision times, and the breakdown threshold for the cumulative loss of man power in the organization are independent and identically distributed exponential random variables. Usha et.al[9] have studied the work in[6] when inter-policy decision times are exchangeable and constantly correlated exponential random variables. In [1] and [2] Dhivya and Srinivasan have extended the work in [6] for a two grade manpower system according as the inter-policy decisions and inter-transfer decisions form the same or different ordinary renewal process respectively. In [3], Dhivya and Srinivasan have studied their work in [1] and [2] using univariate **max** policy of recruitment. In [4], Dhivya and Srinivasan have studied their work in [1] when the policy decisions are classified into two types according to the intensity of attrition. In [5], Dhivya and Srinivasan have studied their work in [1] when inter-policy decision times are exchangeable and constantly correlated exponential random variables. The objective of the present paper is to study the problem of time to recruitment in [5] using univariate Max policy of recruitment.

MODEL DESCRIPTION
Consider an organization taking decisions at random epoch $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…,let X_i be a continuous random variables representing the amount of depletion of manpower(loss of man hours) caused due to the i[th] policy decision in the organization. It is assumed that X_i form a sequence of independent and identically distributed random variables with distribution $G(.)$. Let $\bar{X}_m$ be the maximum loss of manpower due to the first m policy decisions in the organization. For j=1,2,3…,let Y_j be a continuous random variable representing the amount of depletion of manpower in the organization caused due to the j[th] transfer decision. It is assumed that Y_j form a sequence of independent and identically distributed random variables with probability distribution function $H(.)$. Let $\bar{Y}_n$ be the maximum loss of

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manpower in the organization due to the first n transfer decisions. For each i and j, X_i and Y_j are statistically independent. Let $C\ (C > 0)$ be the constant breakdown threshold level for the depletion of manpower in the organization. Let the inter-policy decision times be constantly correlated exchangeable and exponential random variables with distribution $F(.)$, probability density function $f(.)$and parameter a, Let $F_m(.)$be the distribution of the waiting time upto m policy decision times. Let the inter-transfer decision times be independent and identically distributed exponential random variables with distribution $W(.)$, probability density function $w(.)$and mean $\frac{1}{\mu_2}(\mu_2 > 0)$. It is assumed that the two sources of depletion are independent. Let $W_n(.)$ be the n-fold convolution of $W(.)$ with itself. The univariate **Max** policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the maximum loss of man hours in the organization exceeds the constant threshold C.

Let T be the random variable denoting the time to recruitment with distribution L(.),probability density function l(.), Laplace transform $\bar{l}(s)$, mean E(T) and variance V(T). Let $N_P(T)$ and $N_{Trans.}(T)$ be the number of policy decisions and transfer decisions taken until the time to recruitment T respectively. Let $\bar{X}_{N_P(T)}$ and $\bar{Y}_{N_{Trans.}(T)}$be the respective total loss of manpower in $N_P(T)$and $N_{Trans.}(T)$decisions until the time to recruitment T.

MAIN RESULTS

$$P(T > t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\begin{Bmatrix} Probability\ that\ there\ are\ exactly\ m\ policy\ decisions\ and\ n\ transfer \\ decisions\ in\ [0,t) and\ the\ maximum\ loss\ of\ manhours\ due\ to\ m\ policy \\ decisions\ and\ n\ transfer\ decisions\ does\ not\ exceed\ the\ threshold\ C \end{Bmatrix}$$

By using laws of probability and from renewal theory [7],

$$P(T > t) = \sum_{m=0}^{\infty}[F_m(t) - F_{m+1}(t)]\sum_{n=0}^{\infty}[W_m(t) - W_{m+1}(t)]P(\max(\bar{X}_m, \bar{Y}_n) \le C) \qquad (1)$$

where $F_0(t) = W_0(t) = 1$.

Since

$$P(\max(\bar{X}_m, \bar{Y}_n) \le C) = [G(C)]^m[H(C)]^n \qquad (2)$$

From (1), (2) and on simplification we get

$$L(t) = [1 - G(C)]\sum_{m=1}^{\infty}F_m(t)[G(C)]^{m-1} + [1 - H(C)]\sum_{n=1}^{\infty}W_n(t)[H(C)]^{n-1}$$
$$-[1 - G(C)]\sum_{m=1}^{\infty}F_m(t)[G(C)]^{m-1}[1 - H(C)]\sum_{n=1}^{\infty}W_n(t)[H(C)]^{n-1} \qquad (3)$$

From the hypothesis we note that $w_n(t) = \frac{\mu_2{}^n e^{-\mu_2 t}t^{n-1}}{(n-1)!}$. Therefore we find that

$$[1 - H(C)]\sum_{n=1}^{\infty}W_n(t)[H(C)]^{n-1} = 1 - e^{-\mu_2[1-H(C)]t} \qquad (4)$$

From (3) and (4) we get

$$l(t) = \mu_2[1 - H(C)]e^{-\mu_2[1-H(C)]t} + [1 - G(C)]\sum_{m=1}^{\infty}e^{-\mu_2[1-H(C)]t}f_m(t)[G(C)]^{m-1}$$
$$-[1 - G(C)]\mu_2[1 - H(C)]\sum_{m=1}^{\infty}e^{-\mu_2[1-H(C)]t}F_m(t)[G(C)]^{m-1} \qquad (5)$$

The mean and variance of time to recruitment can be computed from (5) and from the result $E(T) = -\left[\frac{d}{ds}[\bar{l}(s)]\right]_{s=0}$ and $E(T^2) = \left[\frac{d^2}{ds^2}[\bar{l}(s)]\right]_{s=0}$. Thus we get

$$E(T) = \frac{1}{\mu_2[1 - H(C)]} - \frac{[1 - G(C)]}{\mu_2[1 - H(C)]}\sum_{m=1}^{\infty}\bar{f}[\mu_2(1 - H(C))][G(C)]^{m-1} \qquad (6)$$

and

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$$E(T^2) = \frac{2}{\left[\mu_2[1-H(C)]\right]^2} - \frac{2[1-G(C)]}{\left[\mu_2[1-H(C)]\right]^2}\sum_{m=1}^{\infty} \bar{f}_m\left[\mu_2[1-H(C)]\right][G(C)]^{m-1} + \frac{2[1-G(C)]}{\mu_2[1-H(C)]}\sum_{m=1}^{\infty} \bar{f'}_m\left[\mu_2[1-H(C)]\right][G(C)]^{m-1} \quad (7)$$

When $K_i, i = 1,2,\dots,m$ are exchangeable and constantly correlated exponential random variables with correlation R, Gurland [8] has obtained the expression for the cumulative distribution function of the partial sum $S_m = K_1 + K_2 + \cdots + K_m$ as

$$P(S_m \le x) = (1-R)\sum_{i=0}^{\infty} \frac{(mR)^i}{(1-R+mR)^{i+1}} \frac{\psi\left(m+i, {}^{x}/_{b}\right)}{(m+i-1)!} \quad (8)$$

where $\psi(n,x) = \int_0^x e^{-u}\, u^{n-1} du$ and $b = a(1-R), a$ being the parameter of the exponential distribution.

Therefore in this paper

$$\bar{f}_m(s) = \frac{1}{(1+bs)^m\left(1+\frac{mRbs}{(1-R)(1+bs)}\right)} \quad (9)$$

Using (9) in (6) and (7), and on simplification we get

$$E(T) = \frac{1}{\mu_2[1-H(C)]} - \frac{[1-G(C)]}{\mu_2[1-H(C)]}\sum_{m=1}^{\infty} \frac{(1-R)\left[1+b\mu_2\big(1-H(C)\big)\right]^{1-m}}{1-R+b\mu_2\big(1-H(C)\big)(1-R+mR)}[G(C)]^{m-1} \quad (10)$$

and

$$E(T^2) = \frac{2}{\left[\mu_2[1-H(C)]\right]^2} - \frac{2[1-G(C)]}{\left[\mu_2[1-H(C)]\right]^2}\sum_{m=1}^{\infty} \frac{(1-R)\left[1+b\mu_2\big(1-H(C)\big)\right]^{1-m}}{1-R+b\mu_2\big(1-H(C)\big)(1-R+mR)}[G(C)]^{m-1} + \frac{2[1-G(C)]b^2(1-R)}{\mu_2[1-H(C)]}\sum_{m=1}^{\infty}\Big(1+b\mu_2\big(1-H(C)\big)\Big)^{-m}\left\{\frac{b\mu_2\big(1-H(C)\big)\big(1-m(1-R+mR)\big)-m}{\left[1-R+b\mu_2\big(1-H(C)\big)(1-R+mR)\right]^2}\right\}[G(C)]^{m-1} \quad (11)$$

Special Case

Suppose X_i *and* Y_j , $i.j = 1,2,3 \dots$,follow exponential distribution with parameters $\alpha_1$and α_2 respectively.
In this case,

$$E(T) = \frac{1}{\mu_2 e^{-\alpha_2 C}} - \frac{e^{-\alpha_1 C}}{\mu_2 e^{-\alpha_2 C}}\sum_{m=1}^{\infty}\frac{(1-R)[1+b\mu_2 e^{-\alpha_2 C}]^{1-m}}{1-R+b\mu_2 e^{-\alpha_2 C}(1-R+mR)}[1-e^{-\alpha_1 C}]^{m-1} \quad (12)$$

and

$$E(T^2) = \frac{2}{[\mu_2 e^{-\alpha_2 C}]^2} - \frac{2e^{-\alpha_1 C}}{[\mu_2 e^{-\alpha_2 C}]^2}\sum_{m=1}^{\infty}\frac{(1-R)[1+b\mu_2 e^{-\alpha_2 C}]^{1-m}}{1-R+b\mu_2 e^{-\alpha_2 C}(1-R+mR)}\left[1-e^{-\alpha_1 C}\right]^{m-1} + \frac{2e^{-\alpha_1 C}b^2(1-R)}{\mu_2 e^{-\alpha_2 C}}\sum_{m=1}^{\infty}(1+b\mu_2 e^{-\alpha_2 C})^{-m}\left\{\frac{b\mu_2 e^{-\alpha_2 C}\big(1-m(1-R+mR)\big)-m}{[1-R+b\mu_2 e^{-\alpha_2 C}(1-R+mR)]^2}\right\}\left[1-e^{-\alpha_1 C}\right]^{m-1} \quad (13)$$

(12) together with (13) give the mean and variance of the time to recruitment for the present case.

Note

Some performance measures related to time to recruitment are presented below.

1. The average number of policy decisions taken until the time to recruitment T is

$$E\big(N_P(T)\big) = \int_0^{\infty} E\big(N_P(t)\big)\, l(t)dt$$

2. The average number of transfer decisions taken until the time to recruitment T is

$$E\big(N_{Trans.}(T)\big) = \int_0^{\infty} E\big(N_{Trans.}(t)\big)\, l(t)dt = \mu_2 E(T)$$

3. The cumulative loss of manpower due to $N_P(T)$ policy decisions is

$$\bar{X}_{N_P(T)} = E(X_i)E\big(N_P(T)\big) = E\big(N_P(t)\big)[E(X_{Ai}) + E(X_{Bi})]$$

4. The cumulative loss of manpower due to $N_{Trans.}(T)$ transfer decisions is

$$\bar{X}_{N_{Trans.}(T)} = E(X_i)E\big(N_{Trans.}(T)\big) = \mu_2 E(T)[E\big(Y_{Aj}\big) + E(Y_{Bj})]$$

5. Hazard rate at T = $\frac{l(t)}{1-L(t)}$

$=P\left(t < T < t + \frac{dt}{T} > t\right) = \frac{L(t+dt)-L(t)}{1-L(t)}$

6. Average residual time for recruitment given that there is no recruitment upto time t.
$= E(T - t/T > t) = \frac{\int_t^\infty [1-L(u)]\,du}{1-L(t)}$

CONCLUSION

The manpower planning model developed in this paper is new in the context of correlated inter-policy decision time. This model can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management levels in the context of attrition. There is a scope for studying the applicability of the designed model using simulation. Further, by collecting relevant data, one can test the goodness of fit for the distributions assumed in this paper. The results given in this paper enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but also in a wider domain.

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